

Cramer's Rule

Introduction

Cramer's rule is a method for solving linear simultaneous equations. It makes use of determinants and so a knowledge of these is necessary before proceeding.

1. Cramer's Rule - two equations

If we are given a pair of simultaneous equations

$$\begin{aligned}a_1x + b_1y &= d_1 \\ a_2x + b_2y &= d_2\end{aligned}$$

then x , and y can be found from

$$x = \frac{\begin{vmatrix} d_1 & b_1 \\ d_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \quad y = \frac{\begin{vmatrix} a_1 & d_1 \\ a_2 & d_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

Example

Solve the equations

$$\begin{aligned}3x + 4y &= -14 \\ -2x - 3y &= 11\end{aligned}$$

Solution

Using Cramer's rule we can write the solution as the ratio of two determinants.

$$x = \frac{\begin{vmatrix} -14 & 4 \\ 11 & -3 \end{vmatrix}}{\begin{vmatrix} 3 & 4 \\ -2 & -3 \end{vmatrix}} = \frac{-2}{-1} = 2, \quad y = \frac{\begin{vmatrix} 3 & -14 \\ -2 & 11 \end{vmatrix}}{\begin{vmatrix} 3 & 4 \\ -2 & -3 \end{vmatrix}} = \frac{5}{-1} = -5$$

The solution of the simultaneous equations is then $x = 2$, $y = -5$.

2. Cramer's rule - three equations

For the case of three equations in three unknowns: If

$$a_1x + b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

$$a_3x + b_3y + c_3z = d_3$$

then x , y and z can be found from

$$x = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}} \quad y = \frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}} \quad z = \frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}$$

Exercises

Use Cramer's rule to solve the following sets of simultaneous equations.

a)

$$7x + 3y = 15$$

$$-2x + 5y = -16$$

b)

$$x + 2y + 3z = 17$$

$$3x + 2y + z = 11$$

$$x - 5y + z = -5$$

Answers

a) $x = 3$, $y = -2$. b) $x = 1$, $y = 2$, $z = 4$